

Performance Evaluation Algorithm for GI/G/m Queues

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Abstract— In this study, we propose an algorithm to evaluate the performance of a GI/G/m queuing network model using a general continuous probability distribution. Queuing models have traditionally been used for quantitative evaluation of packet and phone line congestion is reasonable to use predictions and evaluations using simulation models with stochastic processes. We propose a hybrid algorithm that uses Monte Carlo simulation to generate statistical data with continuous probability distribution. The data is analyzed based on the selected evaluation metric and the length of the queue using simulation and approximate solution methods. The algorithm is suited for environments that require fast verification.

Keywords— GI/G/m, queueing network analyzer, Monte Carlo Method

I. INTRODUCTION

We evaluated the graph events of a GI/G/m queuing network model that uses a non-Markovian general distribution. This model is theoretically difficult to exploit and various approximations have been proposed. The simulated model uses a general distribution, where the distribution between the external / internal arrival rates and the processing time interval are independent of each other. Nodes with multiple receptors also exist, enabling the flow of jobs (or packets) to either branch or merge. Various characteristics of the merging and branching can be calculated by utilizing the queuing network theory based on the in-system flow class (job) departure and arrival variation coefficients, route, and in-stream flow. We built a small queuing network model and verified it with the verification model using the approximate waiting time formula proposed by Sakasegawa [1, 3]. The arrival rate in the external arrival flow was calculated using the Monte Carlo simulation method with a gamma distribution. An approximate solution for the waiting time in each time unit was used as the expected in-system waiting time. Verification using approximate solutions and simulation models have been proposed to be incorporated into mathematical models to evaluate their performance [2, 4, 5, 6].

II. OUTLINE OF MATHEMATICAL MODEL

The event in the model is divided into five nodes, and incomplete jobs (or packets, hereinafter referred to as jobs) are shown as connections between the nodes, representing the flow of jobs. Each node can have multiple contact points to accept jobs, and a job queue processes them at the receptor when the nodes are assigned with jobs. A job may not be returned; this is referred to as a system with no call loss. The utilization rate of each node is ρ_i , and the number of receptors is estimated by fixing the utilization rate to a value less than 100%. The processing time for each node is a random variable that depends on the type of job. The receptors are assigned

independent processing time intervals at each node. They are mutually independent and have the same distribution. A job flows from one node to another along the routes and is processed at each node accordingly. The proposed model is illustrated in Fig. 1.

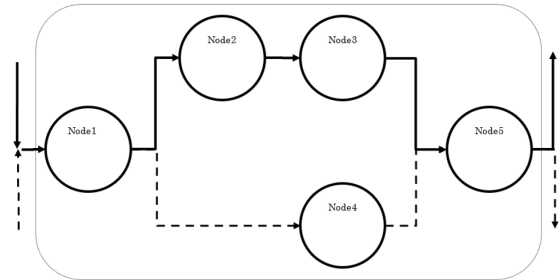


Fig. 1. The open Jackson network model containing five processing nodes (The network interior is shown in the enclosed lines. The straight line represents route 1, and the dotted line represents route 2.)

III. ALGORITHM AND DEFINITIONS OF THE MODEL

Both simulation and approximate computations were considered in the proposed verification model. The simulation generates a random number of probability distributions based on the gamma distribution. The results are continuous parameters, and the outward reaching flow rate is calculated on a continuous time scale. While the model is based on Monte Carlo simulation, it is necessary to use the gamma distribution because the approximation $E(W_{GI/G/m})$ in the evaluation formula is a continuous approximation. Therefore, it is important to note that the time scale and parameters are continuous. Our study also focuses on graphed event-wide productivity. The proposed simulation is based on the algorithm to calculate the approximate waiting time. The probability of the two job streams departing from node 1 to nodes 2 and 4 is assumed to be 1/2.

A. Model Definition

Time: continuous, $t: \{t_0, \dots, t_n\}, n \in \mathbb{Q}_+$

Update: discrete

Distribution of $\widehat{\lambda}_k$ from outside the network: continuous,

$$f(x; d, \theta) = \frac{e^{-x/\theta}}{\Gamma(d)\theta^{d-1}} \quad (for \ x > 0 \text{ and } d, \theta > 0)$$

$\Gamma(d)$ is the gamma function evaluated at d .

d is the shape parameter, and θ is the scale parameter.

Receptors: discrete parameter, $m_i \{m_0, \dots, m_i\}, i \in \mathbb{N}_0$

B. Algorithm:

Impute: Input data predetermined in advance.

$$\text{Solve: } c_{0j}^2 = (1 - \bar{W}_j) + \bar{W}_j \left[\sum_{k=1}^r c_k^2 \left(\frac{\widehat{\lambda}_{11}\{k:n_{kl}=j\}}{\sum_{l=1}^r \widehat{\lambda}_{11}\{k:n_{kl}=j\}} \right) \right],$$

$$\bar{W}_j \equiv \left[1 + 4(1 - \rho_j)^2 (\bar{v}_j - 1) \right]^{-1},$$

$$\text{and } \bar{v}_j = \left[\sum_{k=1}^r \left(\frac{\widehat{\lambda}_k \mathbf{1}\{k:n_{kl}=j\}}{\sum_{l=1}^n \widehat{\lambda}_l \mathbf{1}\{l:n_{kl}=j\}} \right)^2 \right]^{-1}$$

Generate: Gamma distribution $(\widehat{\lambda}_k, c_{0j}^2)$

$$\text{Solve: } \lambda_{0j} = \sum_{k=1}^r \widehat{\lambda}_k \mathbf{1}\{k:n_{kl}=j\}$$

$$\text{Solve: } \tau_j = \frac{\sum_{k=1}^r \sum_{l=1}^n \widehat{\lambda}_k \tau_{kl} \mathbf{1}\{(k,l):n_{kl}=j\}}{\sum_{k=1}^r \sum_{l=1}^n \widehat{\lambda}_k \mathbf{1}\{(k,l):n_{kl}=j\}}$$

$$\text{Solve: } \lambda_j = \lambda_{0j} + \sum_{i=1}^n \lambda_i \gamma_i q_{ij}$$

$$\text{Solve: } \lambda_{ij} = \lambda_i q_{ij}$$

$$\begin{cases} \text{if } \widehat{\lambda}_1 > \widehat{\lambda}_2, q_{ij} = q_{14} = \widehat{\lambda}_2 / \widehat{\lambda}_1, q_{12} = 1 - \widehat{\lambda}_2 / \widehat{\lambda}_1 \\ \quad \text{if } \widehat{\lambda}_1 = \widehat{\lambda}_2, q_{ij} = q_{14} = 0.5 \\ \text{if } \widehat{\lambda}_1 < \widehat{\lambda}_2, q_{ij} = q_{12} = \widehat{\lambda}_1 / \widehat{\lambda}_2, q_{14} = 1 - \widehat{\lambda}_1 / \widehat{\lambda}_2 \end{cases}$$

Solve: m_i using $\rho_i = \lambda_i \tau_i / m_i, 1 \leq i \leq n$ with utility ratio less than 100%

$$\text{Solve: } \tau_j^2 (c_{s_j}^2 + 1) = \frac{\sum_{k=1}^r \sum_{l=1}^n \widehat{\lambda}_k \tau_{kl}^2 (c_{s_{kl}}^2 + 1) \mathbf{1}\{(k,l):n_{kl}=j\}}{\sum_{k=1}^r \sum_{l=1}^n \widehat{\lambda}_k \mathbf{1}\{(k,l):n_{kl}=j\}}$$

$$\text{Solve: } c_{a_i}^2 = 1 + (1 - \rho_i^2)(c_{a_i}^2 - 1) + \frac{\rho_i^2}{\sqrt{m}} (\max\{c_{s_i}^2, 0.2\} - 1)$$

$$\text{and } c_i^2 = p_i c^2 + 1 - p_i$$

$$\text{Solve: } c_H^2 = w \sum_i \left(\frac{\lambda_i}{\sum_k \lambda_k} \right) c_i^2 + 1 - w,$$

$$w = [1 + 2.1(1 - \rho)^{1.8} v]^{-1}, \text{ and } v = \left[\sum_i \left(\frac{\lambda_i}{\sum_k \lambda_k} \right)^2 \right]^{-1}$$

$$\text{Solve: } E(W_{GI/G/m}) = \left(\frac{c_{a_i}^2 + c_{s_i}^2}{2} \right) \left(\frac{\rho_i \sqrt{2(m_i+1)-1}}{m_i(1-\rho_i)} \tau_i \right)$$

Solve: $\sum_{j=1}^{n_k} E(Wn_{kj})$, and advance one-unit time

The Monte Carlo simulation, which uses a generated gamma distribution, is executed with the unit time advancing by one per step. The receptor number at each node is calculated according to the formula $\rho_i = \lambda_i \tau_i / m_i, 1 \leq i \leq n$. The utilization rate is set to less than 1 in advance, and the number of receptors m_i is estimated to m_i a rounded integer value. This algorithm describes the sequence of events of a process that is simulated iteratively per unit time. For additional information on the parameters, please refer to the notation list.

IV. DISCUSSIONS

This algorithm was verified to work well with Monte Carlo simulation by using the approximation formula for general distribution. It is also possible to utilize our algorithm for general computational tasks. However, the nature of approximations for general distributions is difficult when solving for multiple variables. In the future, we hope to simplify the approximation formula further for GI/G/m queueing network models.

NOTATION LIST

$c_{a_i}^2, c_{s_i}^2$: Squared variation coefficient of the arrival, and processing time interval of a job at node i
c_{0j}^2 : Squared variation coefficient of the arrival interval of the flow arriving at node j from outside the network
$c_{s_{kj}}^2$: Squared variation coefficient of the processing time interval for the j-th process on route k
c_k^2 : Squared variation coefficient of the arrival interval when the job for class k arrives from outside

$c_{d_i}^2$: Squared variation coefficient of the departure time interval of the job flow leaving for the next node after processing at node i
c^2 : Squared variation coefficient before the job flow leaves for the subsequent node branches after processing at node i
c_i^2 : Squared variation coefficient of the i-th branch flow
c_H^2 : Squared variation coefficient of the arrival time interval after joining node j
$c_{w_i}^2$: Squared variation coefficient of the waiting time W when the window at node i is congested
EW: Expected waiting time
$E(W_{GI/G/m})$: Expected waiting time in queue GI/G/m
$E(Wn_{kj})$: Expected waiting time at the j-th node on path k
m_i : Receptors at node i
n_{kl} : l-th node n_l on route k, and n_{ll} : l-th node n_l on path l, where the flow arriving from route (job) k exists
q_i : Selected branch probability ($i = 1, 2, \dots, k$ is flow number)
q_{ij} : Probability of flow branching
λ_i : Internal arrival flow rate per unit time at node i
λ_{ij} : Flow rate per hour between nodes i and j
$\widehat{\lambda}_k$: Estimated flow rate per hour when jobs for each route k arrive at the network from outside
μ_i : Amount of jobs per hour, average processing rate
τ_i : Processing time of each node i
τ_{kj} : Processing time at the j-th node on route k
ρ_i : Utilization ratio of processing at node i
γ_i : Job generation variable at node i (If no job is generated at a node in this model, it is set to 1)
$w, (\bar{W}_j)$: The approximate correction rate of ρ, ρ_j (average)
$v_j, (\bar{v}_j)$: The approximate correction rate in the intra-system job flow at node j (average)
v: The approximate correction rate of the job flow
p_i : This is the branch probability for the branching flow. In this study, it is set to the same value as q_i

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