

# Laplace Equation-Based High-Speed Autonomous Clustering for MANET

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**Abstract**—Autonomous decentralized clustering technologies to realize hierarchical route control for mobile ad hoc network terminals have been studied. The autonomous decentralized clustering must be able to realize clustering that can adapt to various movement patterns of terminals. Particularly, it must be effective even when user density is very high, such as around a train station during rush hour. However, existing methods have found that the cluster structure cannot be maintained when user density becomes very high. This paper proposes an autonomous decentralized clustering method that can maintain the cluster structure even if user density is high. We introduce an autonomous decentralized algorithm based on the Laplace equation that enables the method to quickly respond to user movement.

**Index Terms**—MANET, autonomous decentralized clustering, Laplace equation, high-speed clustering

## I. INTRODUCTION

The significant characteristic of MANETs is that the terminals (hereinafter, nodes) communicate with other nodes that are within the radio transmission range and transfer data to the target node by multi-hop communications. MANETs are expected to be used for emergency communication when a base station becomes unavailable due to a large-scale disaster.

Routing is a vital MANET function since nodes move, and the simplest routing, flooding, has poor scalability as the number of control packets for route search increases exponentially with network size. To solve this problem, it is necessary to introduce clusters. Limiting the flooding range to within clusters improves scalability. In MANET, since each node can access only the local information that can be acquired from itself and neighboring nodes, clusters must be generated in an autonomous distributed manner based only on local information. This function is called autonomous decentralized clustering [1].

Because MANET nodes move often, it is necessary to design autonomous clustering to suit the various movement characteristics of nodes. However, the conventional clustering based on the diffusion equation [2] has the problem that clusters are combined when the node density becomes high [3]. To solve this problem, we need a method that can form many clusters in an area with high node density without using the information of node density.

In this paper, we propose autonomous clustering based on the Laplace equation to adapt to dynamic environments where the node density changes. The effectiveness of the proposed method is demonstrated by simulations in a dynamic environment.

The rest of this paper is organized as follows. Section II overviews autonomous decentralized clustering based on the Laplace equation and then proposes an acceleration technique of the diffusion effect. Section III shows the effectiveness of the proposed method by comparing it to existing methods in a dynamic environment. Finally, Section IV describes the conclusions and future work.

## II. CLUSTERING BASED ON THE LAPLACE EQUATION

### A. Clustering based on the diffusion equation

This subsection explains the autonomous decentralized operation rules of each node for autonomous clustering based on the diffusion equation. This is the basic technology underlying autonomous clustering based on local information.

For a certain node  $i$ , let  $\partial i$  be the set of adjacent nodes of  $i$ , and let  $\Delta t$  be the time interval of autonomous operations for each node. Let  $q_i(t_k)$  be the state quantity of node  $i$  at time  $t_k = 1, 2, \dots$  where  $t_{k+1} - t_k = \Delta t$ . Then, the temporal evolution of  $q_i(t_k)$  is given as

$$q_i(t_{k+1}) = q_i(t_k) - \Delta t \sum_{j \in \partial i} J_{i,j}^{\text{diff}}(t_k), \quad (1)$$

where  $J_{i,j}^{\text{diff}}(t_k)$  represents the amount of movement per unit time that the state quantity moves from node  $i$  to node  $j$  due to the effect of diffusion.  $J_{i,j}^{\text{diff}}(t_k)$  is expressed as

$$J_{i,j}^{\text{diff}}(t_k) = -\frac{\sigma^2}{d_i + 1} (q_j(t_k) - q_i(t_k)), \quad (2)$$

where  $d_i$  denotes the nodal degree of node  $i$  and  $\sigma^2$  is a positive parameter that determines the strength of diffusion. Note that it is necessary to set  $\sigma^2 \leq 1$  so that the state quantity does not become negative. In (2), the diffusion coefficient is  $\sigma^2/(d_i + 1)$ , and the reason for making it dependent on the nodal degree is to reduce the variation in diffusion speed over the network. State quantity  $q_i$  of node  $i$  is successively updated by the diffusion effect.

The procedure to determine the cluster heads (CHs) and cluster structure based on state quantity  $q_i(t_k)$  of each node at time  $t_k$  is as follows. Each node compares its state quantity with those of the neighboring nodes, selects the node having the largest state quantity from among the neighboring nodes. If the state quantities are the same, the node with the larger node number is determined as the node with the larger state quantity. By following nodes which has the maximum state quantity, it reaches the node at the maximum point and it is taken to be a CH. Also, all nodes that have reached the same CH by the node selection operation belong to the same cluster.

In addition to the above-mentioned rules of operation, there are techniques for cluster construction that retain past state quantities as history information represented by a spatial structure vector [4]. We treat this method as a comparison to the proposed method in Chapter III. The spatial structure vector consists of  $n$  ( $n = 0, 1, \dots, N$ ) components, and for each component of the spatial structure vector at each operating time, the amount of state at the next time is calculated based on diffusion, and the  $n$  component is shifted to the  $N + 1$  component. Since it effectively stops the time evolution of the states, we can control the number of clusters by changing the components of the states used for clustering.

### B. Clustering based on the Laplace equation

As described in Section I, it is necessary to design autonomous clustering so that it can operate properly even in environments where user density changes. Therefore, [5] proposed autonomous clustering based on the Laplace equation assigning a specified node (hereafter, SN) to be a CH. Since it is expected that many CHs exist in an area with high node density, cluster merger can be prevented even in an environment with high user density.

Details of control rules are described below. The SN is given a higher initial value of  $q_{ch}$  than the other nodes so that it is CH from the beginning. In addition, to prevent to reduction the state quantity of SN by the effect of diffusion, we apply a mechanism to keep the state quantity at  $q_{ch}$  by supplying the reduced state quantity.

Next, we will explain the rules for nodes other than SNs. Since the state quantity of the SN always maintains the initial value of  $q_{ch}$  by supplying the reduced quantity from outside of the network, the total amount of state quantity is increased. Therefore, if there is no mechanism for draining the state quantity of nodes to outside the network, the state quantities of all nodes will approach  $q_{ch}$  with time by the diffusion effect, and cluster structure is lost. To avoid this problem, (1) is rewritten as

$$q_i(t_{k+1}) = \left( q_i(t_k) - \Delta t \sum_{j \in \partial i} J_{i,j}^{\text{diff}}(t_k) \right) (1 - r_{\text{dec}}). \quad (3)$$

So, each node decreases its state quantity at the constant rate of  $r_{\text{dec}}$  when updating state quantity.

TABLE I  
VALUE OF EACH PARAMETER

parameters	$r_{\text{dec}}$	$q_{\text{ch}}$	$\sigma^2$	Time
value	$0.3 \leq r_{\text{dec}} \leq 0.9$	3.0	1.7	300

### C. Acceleration of the diffusion effect

As mentioned in Section I, cluster configuration speed is an important factor in implementing autonomous clustering in a dynamic environment. Since the method described in Section II-B faces a barrier when enhancing clustering speed, this section proposes an acceleration technique of the diffusion effect that overcomes the barrier.

The speed of the diffusion is controllable by adjusting the value of the parameter  $\sigma^2$ . To increase the cluster configuration speed, one approach is to increase the value of the diffusion coefficient. However, in equations (1) and (2), there is a problem that if the value of  $\sigma^2$  is larger than 1, the state quantity becomes negative. To avoid this issue, if the value of  $\sigma^2$  is greater than 1 and the state quantity becomes negative, we implement a new control rule that prevents the state quantity from becoming negative. If the state quantity based on the control rules (1) and (2) will be negative, the node  $i$  moves a portion of quantity proportional to the rates  $J_{i,j}^{\text{diff}}(t_k)$  and make  $q_i(t_{k+1})$  to 0.

This new control rule ensures that the value of the diffusion coefficient is larger than is possible with the existing method, which will increase the clustering speed.

In addition, we also propose to adjust the value of  $r_{\text{dec}}$ . While  $r_{\text{dec}}$  is a parameter that determines the decay rate of the state quantity, we have found that setting it to an appropriate value can further accelerate the diffusion effect.

## III. EXPERIMENT IN A DYNAMIC ENVIRONMENT

### A. Experiment in random dynamic environment

First, we show experimental results for constant user density by using a random walk model for node movement. There are four SNs and 496 nodes in the two-dimensional space of  $[0, 1] \times [0, 1]$ . The initial positions of these nodes are randomly distributed. These nodes are moving at random directions and speeds, form a link between nodes that exist within a certain distance.

The parameters used in the experiment are shown in Table I. The experimental results shown below are the averages of the experimental results for the five network models.

We perform an evaluation to compare the proposed method with the existing method in the random dynamic environment. The evaluation shows the number of CHs with respect to time. The time transition of the number of CHs in the proposed method is shown on the left of Fig. 1, and the result of the existing method is shown on the right. Here, the legend on the left shows the value of  $r_{\text{dec}}$ , and the legend on the right shows the position of the component used in the state vector (the time of diffusion stops). The results of the proposed method show that the number of CHs is more than the number of SNs. This

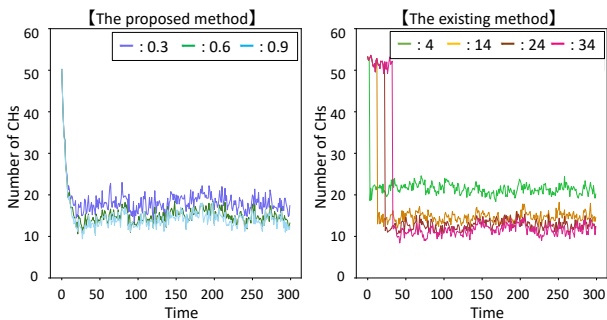


Fig. 1. CH transition over time in a random dynamic environment

is caused by the disconnection of the network and transient clustering situations. In addition, the number of CHs can be adjusted by manipulating the value of  $r_{dec}$ . On the other hand, the results of the existing methods show that the number of CHs can be adjusted by generating clusters according to the network conditions and changing the time of diffusion stops, as in the proposed method. Furthermore, it can be seen that the proposed method is superior in that it offers adaptability to the random dynamic environment and can control the number of clusters without having historical information, unlike the existing method. Since no historical information is needed, it is possible to form clusters to some extent at Time = 5, which means that new clusters can be formed more quickly.

#### B. Experiment in high-density and random dynamic environment

This subsection describes an experiment in random dynamic environment with user density variations. Conventional autonomous clustering based on the diffusion equation has problem of cluster merger under high-density environments. Therefore, we evaluate whether the proposed method can cope with high-density and random dynamic environments.

A high-density environment is a situation in which the number of adjacent nodes within the communicable distance increases due to increase in nodes density. Therefore, in this simulation experiment, the maximum communication distance (0.07 above) is increased gradually up to 0.14 (node density is quadruple). To match the node movement speed with the scale of the communication distance, node speed is also gradually doubled. All other conditions are the same as in Section III-A. This setting yields a random dynamic environment in which nodes move by randomly and the user density is changes to higher.

We perform evaluations to compare the proposed method with the existing method in the high-density and random dynamic environment as in Section III-A. The results of the proposed method show that when the node density increases with time, the number of CHs takes a value close to the number of SNs 4. This is because the state quantity of  $q_{ch}$  is kept at a relatively large value which ensures SNs are likely to remain CHs. Therefore, the number of clusters is guaranteed to be at least close to the number of SNs. On the other hand,

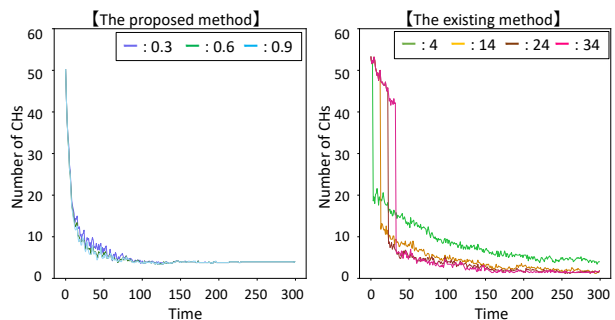


Fig. 2. CH transition over time in a high-density and random dynamic environment

the existing method allows the number of CHs to decrease, clusters are merged, when node density increases. Therefore, it is clear that the existing method cannot cope with high-density environments.

#### IV. CONCLUSION

In this paper, we proposed a high-speed MANET clustering method for autonomous clustering based on the Laplace equation. The results of the comparative evaluations in two dynamic environments showed that the proposed method has faster cluster configuration speed and can better cope with various movement models including node density changes compared to the existing method.

As future work, we plan to evaluate the characteristics of an appropriate combination with the CH autonomous generation method [3].

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